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Author(s): Jang-Ting Guo and Federico Sturzenegger
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CRAZY EXPLANATIONS OF INTERNATIONAL BUSINESS CYCLES*

BY JANG-TING GUO AND FEDERICO STURZENEGGER[†1]

University of California at Riverside, U.S.A.
University of California at Los Angeles, U.S.A.

International real business cycle models have been unable to provide a good explanation for the *consumption-output anomaly*: in theoretical economies, consumption is more strongly correlated across countries than is output, whereas the opposite is the case in the data. This paper examines an increasing returns-to-scale model in which the economy is subject to 'belief' shocks that affect the consumption Euler equations rather than productivity. Under the assumption that there are no contingent claim markets on the realizations of 'sunspots,' the belief-driven model can account for the consumption-output anomaly even with a separable period utility function.

1. INTRODUCTION

Since Backus et al. (1992), many authors have attempted to adopt the two-country version of dynamic general equilibrium models to explain the properties of international real business cycles (IRBC). Studies in this line of research have produced a set of stylized facts as well as clearly identified problems. One of the most robust puzzles is what has been dubbed the *consumption-output anomaly*. It refers to the fact that in the data, outputs across countries are more strongly correlated (0.70) than are consumptions (0.46), while the opposite holds in theoretical economies. In standard IRBC models with a constant returns-to-scale (CRS) technology and perfect competition, the assumption of complete markets on the realizations of shocks to economic fundamentals like productivity disturbances induce an important degree of risk diversification. When risks are properly diversified, consumption levels in both countries move together as functions of aggregate (world) productivity, which in turn implies high cross-country consumption correlation. Moreover, productivity shocks inducing capital flows across countries together with labor supply adjustments within each country lead to negative international output correlation.

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[†] E-mail: *guojt@mail.ucr.edu; sturzenegger_f@impsat1.com.ar*

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One way to deal with the consumption-output anomaly was suggested by Stockman and Tesar (1995), who consider an economy with nontraded goods subject to preference shocks. The introduction of nontradable consumption reduces the cross-country correlation of aggregate consumption because agents do not have incentives to trade claims on the output of the nontraded goods sector. Alternatively, it has been shown that incomplete asset or commodity markets may help resolve the puzzle. For example, Baxter and Crucini (1995) and Kollmann (1996) both examine an economy in which agents trade a single risk-free bond. While Kollmann finds low consumption correlations, output remains less correlated across countries than consumption. Baxter and Crucini show that when the model is driven by difference stationary productivity shocks (rather than the more standard trend stationary specification), output correlations can exceed consumption correlations between countries, but their reported international consumption correlation is negative, which is not observed in the data.

This paper builds on the incomplete markets literature, with a slightly different approach. We introduce increasing returns-to-scale, which are standard in the international trade literature, into the traditional IRBC economy. It is well known that increasing returns can have significant effects in a real business cycle (RBC) model. Baxter and King (1991) have shown that increasing returns improve the empirical performance of an otherwise standard RBC model for the closed economy. In addition, Benhabib and Farmer (1994) demonstrate that if the degree of increasing returns is strong enough, the steady state may lose its saddle point stability, and turn into an *indeterminate* sink with multiple equilibrium paths converging towards it. In such an economy, Farmer and Guo (1994) illustrate that 'animal spirits' or expectational shocks that affect the consumption Euler equation can be an important factor in accounting for U.S. business cycle fluctuations.[2] In this paper we extend the Benhabib and Farmer approach to an international context.

Our main finding is that introducing increasing returns to an economy with complete or incomplete contingent claim markets on technology shocks does not improve the fit of an IRBC model, provided the steady state remains as a saddle point. However, when the increasing returns are strong enough to generate indeterminacy of the equilibrium, under the assumption that there are no markets for claims contingent on 'sunspot' states, the consumption-output anomaly disappears even with a separable period utility function in consumption and leisure. In our baseline economy driven solely by 'belief' shocks, the output correlation across countries is 0.98, while the consumption correlation is 0.44.

[2] The term 'animal spirits' was first introduced by Keynes (1936) and has been resuscitated by Howitt and McAfee (1988, 1992). Since the paper by Azariadis (1981), there has been a large literature exploring the relation between 'animal spirits' and dynamic general equilibrium models. Representative work in this area include Cass and Shell (1983), Farmer and Woodford (1984), Woodford (1991), and Blanchard (1993), among others. In the paper, we use the terms 'animal spirits,' 'sunspots,' and 'self-fulfilling beliefs' interchangeably. All refer to any randomness in the economy that is not related to uncertainty about economic fundamentals such as technology, preferences, and endowment.

The intuition for our findings derives from the fact that these belief shocks affect the consumption Euler equation rather than the production function. As a result, agents' changing beliefs will lead to fluctuations in consumption and labor supply by affecting their desire to spend in each country. A domestic belief shock that increases labor supply induces a higher rate of return to capital, which in turn triggers capital inflows. The foreign country, at the same time, suffers from a capital outflow that is compensated with additional labor employment. Hence labor hours in both countries move in the same direction, inducing a positive cross-country output correlation. In contrast, consumption levels will follow divergent paths, exhibiting low international correlations because of incomplete sunspot markets.

The paper is organized as follows. Section 2 describes the environment of the economy. Section 3 examines equilibria for alternative specifications of the model, depending on the degree of increasing returns-to-scale and driving shocks. Section 4 discusses the calibration of the model. Simulation results are presented in Section 5. Section 6 concludes.

2. THE ENVIRONMENT

2.1. *Firms.* Benhabib and Farmer (1994) have shown that there are two alternative production structures that can reconcile a competitive theory of income distribution with increasing returns in a RBC model. The first is by assuming that there are positive productive externalities that are external to firms. This specification maintains the assumption of perfect competition, and was used by Baxter and King (1991) to evaluate the business cycle implications of increasing returns in a closed RBC economy. The other postulates that some firms are monopolistic competitors, and use a technology with internal increasing returns-to-scale. Both formulations end up with exactly the same expression for the aggregate production function. In addition, the degree of increasing returns required to generate a reasonable fit of business cycle properties is well within the estimates from micro data for both versions of the closed economy. In our exposition of the model, we follow Farmer and Guo (1994) by presenting the version with monopolistic competition.

The world economy is composed of two equally sized countries. For brevity, in this subsection all variables are denoted without explicitly referring to period t. In each country, a unique final consumption good Y is produced from a continuum of intermediate inputs $X(i)$, where $i \in [0,1]$. We assume that each firm uses a Dixit-Stiglitz technology to produce the final good:

$$Y = \left(\int_0^1 X(i)^{\lambda} \, di \right)^{1/\lambda}, \tag{1}$$

where $\lambda \in (0,1)$. The final goods sector is assumed to be perfectly competitive. If we denote $P(i)$ as the relative price of the i'th intermediate input in terms of the final good, then the profit function of a final goods producer is given by:

$$\Pi = Y - \int_0^1 P(i) X(i)\, di. \tag{2}$$

First-order conditions for profit maximization lead to the following demand function for the intermediate good $X(i)$:

$$X(i) = P(i)^{\frac{1}{\lambda - 1}} Y. \tag{3}$$

All intermediate goods are assumed to be produced by the *same* technology using capital and labor as inputs:

$$X(i) = ZK(i)^{\alpha} L(i)^{\beta}, \tag{4}$$

where Z is an aggregate technology shock with unit mean. We introduce increasing returns to the model by allowing $\alpha + \beta$ to be greater than one. Intermediate goods producers are postulated to be monopolistic competitors, each of which exhibits a degree of monopoly power represented by the parameter λ. When $\lambda = 1$, all intermediate inputs are perfect substitutes, thus the model collapses to a competitive economy. Using (3) and (4), the profits for the intermediate goods producer i are:

$$\Pi(i) = Y^{1-\lambda} Z^{\lambda} K(i)^{\alpha\lambda} L(i)^{\beta\lambda} - rK(i) - wL(i), \tag{5}$$

where r is the real rental rate of capital, and w denotes the real wage rate. Notice that even though the production function (4) displays increasing returns, the above profit maximization problem is well defined at the level of individual firms as long as $\lambda(\alpha + \beta) \leq 1$. This implies that economies of scale are ruled out when $\lambda = 1$ (the perfectly competitive case), but the model is consistent with increasing returns of some extent for smaller values of λ.

Maximization of (5) yields the following first-order conditions:

$$\frac{\lambda \alpha X(i) P(i)}{K(i)} = r, \tag{6}$$

$$\frac{\lambda \beta X(i) P(i)}{L(i)} = w. \tag{7}$$

Since symmetry is assumed, we are looking for a solution in which $L(i) = L$, $K(i) = K$, and $P(i) = P$, for all i. Plugging the demand function for intermediate inputs (3) into the zero profit condition of the final goods sector leads to $P(i) = 1$ in equilibrium. Using the symmetry assumption, we can substitute (4) into (3) to obtain the aggregate production function for the final output:

$$Y = ZK^{\alpha} L^{\beta}. \tag{8}$$

It follows that in a symmetric equilibrium, $rK = aY$ and $wL = bY$, where $a \equiv \lambda\alpha$ and $b \equiv \lambda\beta$. That is, the parameter a represents the capital share, whereas b denotes the labor share of total output. As a result, the profit share of national income is equal to $(1 - a - b)$.

2.2. *Households.* We follow Backus et al. (1992) by assuming that both countries produce the same final consumption good. The representative household in each country maximizes the expected utility over its lifetime:[3]

$$U = \sum_{t=0}^{\infty} \rho^t \sum_{z_t \in H_t} \pi(z_t) \left\{ \frac{c_t^{1-\theta}(z_t)}{1-\theta} - \frac{L_t^{1-\gamma}(z_t)}{1-\gamma} \right\}, \qquad \theta > 0 \text{ and } \theta \neq 1, \tag{9}$$

where $\rho \in (0,1)$ is the discount factor, z_t represents a particular realization of the state of nature, $\pi(z_t)$ is the probability of state z_t, and H_t denotes all the possible histories for the realization of this uncertainty.[4] c_t represents consumption by the agent in period t. The parameter θ captures the curvature of the utility function with respect to consumption, and denotes the intertemporal elasticity of substitution. L_t represents labor hours, and $\gamma(\leq 0)$ is the negative inverse of labor supply elasticity. In the benchmark specification with complete markets, the budget constraint faced by the household in each country is:

$$\sum_{t=0}^{\infty} \sum_{z_t \in H_t} q_t \{ c_t + K_{t+1} - (1-\delta)K_t + p_t^1 s_{t+1}^1 + p_t^2 s_{t+1}^2 - r_t K_t - w_t L_t - (p_t^1 + d_t^1)s_t^1 - (p_t^2 + d_t^2)s_t^2 \} = 0, \tag{10}$$

where q_t denotes the time-0 price of one unit of consumption in state z_t, and s_t^i denotes the shares of equity holdings to a firm in country i at time t. Moreover, d_t^i represents the dividends, and p_t^i is the share price of the firms within country i in state z_t measured in units of consumption of that state. Total supply of equities for firms in each country is set to be one. Agents can purchase state-contingent commodities and equities for every moment in time. It is well known that when the period utility function is separable in consumption and leisure, the solution to this problem is characterized by perfect risk pooling. For the current problem, this corresponds to a solution in which $s_t^i = 1/2$, for all i and t, that is, both agents own half the firms in each country.

As in Backus et al. (1994), we can define the spot prices as $Q_t(z) = q_t(z)/\rho^t \pi(z_t)$. The problem then separates into a number of identical sub-problems, one for each realization of the state z_t. The first-order conditions for these problems require that

[3] Unlike Backus et al. (1992) and other earlier studies, we choose a separable utility function in consumption and leisure to highlight that the source of our results comes from indeterminacy of the equilibrium. This utility function implies that consumptions are perfectly correlated across countries in models with complete markets.

[4] We use the notation z to indicate that uncertainty arises from the evolution of productivity shocks. All variables in the equations to follow depend on the realization of z_t, but we omit this dependency to avoid notational confusion.

the marginal utility of consumption be equal to its spot price at each moment in time and for every state. Because all agents face the same prices, this in turn implies that consumptions are perfectly correlated across countries.

Together with the conditions for the optimal allocation of labor and capital, the set of first-order conditions can be reduced, for country $i = 1, 2$, to

$$\frac{c_{it}^{\theta}}{L_{it}^{\gamma}} = b\frac{Y_{it}}{L_{it}}, \tag{11}$$

and

$$c_{it}^{-\theta} = \rho E_t\{c_{it+1}^{-\theta}(1-\delta+r_{it+1})\} = \rho E_t\left\{c_{it+1}^{-\theta}\left(1-\delta+a\frac{Y_{it+1}}{K_{it+1}}\right)\right\}, \tag{12}$$

where (11) equates the marginal rate of substitution between consumption and leisure to the marginal product of labor, and (12) is the standard stochastic consumption Euler equation.

2.3. *Steady State and Dynamics.* To complete the system, we add the economy-wide market clearing condition for the final consumption good, that is, the world budget constraint:

$$K_{t+1} = Y_{1t} + Y_{2t} + (1-\delta)K_t - c_{1t} - c_{2t}, K_0 = \overline{K}_0, \tag{13}$$

where $K_t = K_{1t} + K_{2t}$ is world capital stock. To generate artificial time series from the model, we use equation (8) to eliminate Y_1 and Y_2, and the labor market equilibrium condition (11) to eliminate L_1 and L_2 from (12) and (13). Finally, we assume that the capital stock is fully mobile within a period, which implies that rental rates will be equalized across countries, $r_{1t} = r_{2t}$, for all t.[5] Using this equality, we can substitute away K_1 and K_2 from the world budget constraint (13). This results in a nonlinear system of five dynamic equations in K_t, c_{1t}, c_{2t}, and the two technological shocks Z_{1t} and Z_{2t}:

$$\begin{aligned} K_{t+1} = {} & AZ_{1t}^{m}K_t^{\phi}c_{1t}^{d}(1+Z_{1t}^{p}Z_{2t}^{-p}c_{1t}^{s}c_{2t}^{-s})^{-\phi} \\ & + AZ_{2t}^{m}K_t^{\phi}c_{2t}^{d}(1+Z_{1t}^{p}Z_{2t}^{-p}c_{1t}^{-s}c_{2t}^{s})^{-\phi} + (1-\delta)K_t - c_{1t} - c_{2t}, \end{aligned} \tag{14}$$

$$c_{1t}^{-\theta} = E_t\{BZ_{1t+1}^{m}K_{t+1}^{\phi-1}c_{1t+1}^{d-\theta}(1+Z_{1t+1}^{p}Z_{2t+1}^{-p}c_{1t+1}^{s}c_{2t+1}^{-s})^{-\phi+1} + \tau c_{1t+1}^{-\theta}\}, \tag{15}$$

$$c_{2t}^{-\theta} = E_t\{BZ_{2t+1}^{m}K_{t+1}^{\phi-1}c_{2t+1}^{d-\theta}(1+Z_{1t+1}^{-p}Z_{2t+1}^{p}c_{1t+1}^{-s}c_{2t+1}^{s})^{-\phi+1} + \tau c_{2t+1}^{-\theta}\}, \tag{16}$$

[5] This specification is unusual in that standard IRBC models allow for investment, rather than capital stocks, to be mobile. As a result, the negative output correlations across countries in four out of five models that we consider are more extreme than usual. While this gives quantitatively different results from previous studies, we can still get a feeling for the contribution of sunspots by using the same specification for all cases.

$$(17) \qquad \begin{bmatrix} Z_{1t+1} \\ Z_{2t+1} \end{bmatrix} = \Omega \begin{bmatrix} Z_{1t} \\ Z_{2t} \end{bmatrix} + \begin{bmatrix} \eta_{1t+1} \\ \eta_{2t+1} \end{bmatrix}, Z_{10} = \bar{Z}_1, \text{ and } Z_{20} = \bar{Z}_2,$$

where

$$\omega \equiv 1/\beta + \gamma - 1, \quad m \equiv 1 - \omega\beta, \quad A \equiv (1/b)^{\omega\beta}, \quad \phi \equiv \alpha m, \quad p \equiv m/(\phi - 1),$$
$$d \equiv \theta\omega\beta, \quad s \equiv d/\phi - 1, \quad B \equiv \rho a A, \quad \text{and} \quad \tau \equiv \rho(1 - \delta).$$

Following Backus et al. (1992), equation (17) specifies the productivity disturbances for the two countries (Z_1 and Z_2) as a bivariate autoregressive process. In the models where productivity shocks are the driving force for business cycle fluctuations, we scale the estimated Solow residuals such that the mean of Z_i is equal to one in country i. The innovation η_i is assumed to be an *i.i.d.* random variable with bounded support and standard deviation σ_{η_i}.

Since physical capital is mobile in each period, the marginal product of capital will be identical across countries. This implies that:

$$(18) \qquad K_{1t} = \frac{K_t}{1 + Z_{1t}^{p} Z_{2t}^{-p} c_{1t}^{s} c_{2t}^{-s}}, \quad \text{and} \quad K_{2t} = \frac{K_t}{1 + Z_{1t}^{-p} Z_{2t}^{p} c_{1t}^{-s} c_{2t}^{s}}.$$

Employment in country i is determined by the static equation:

$$(19) \qquad L_{it} = \left[\frac{1}{b} \frac{c_{it}^{\theta}}{Z_{it} K_{it}^{\alpha}} \right]^{\omega}, \qquad i = 1,2.$$

To analyze the short-run dynamics of alternative models, we take a first-order Taylor series approximation to the dynamic system (14)–(17) around the symmetric interior stationary state of the nonstochastic economy.[6] The steady state is defined by:

$$(20) \qquad K^* = \left(\frac{\mu}{2\nu} \right)^{1/(\chi - 1)}, c_1^* = c_2^* = \nu(K^*)^{\chi}, \text{ and } Z_1^* = Z_2^* = 1,$$

where

$$\mu \equiv \frac{A(1 - \tau)}{B} - \delta, \quad \upsilon \equiv \left[\frac{2^{\phi - 1}(1 - \tau)}{B} \right]^{1/d}, \quad \text{and} \quad \chi \equiv \frac{1 - \phi}{d}.$$

3. PRODUCTIVITY OR EXPECTATIONAL SHOCKS?

3.1. *The Real Business Cycle Benchmark.* The above dynamic system represents the most 'generalized' version of our model economy, which will be used to analyze five alternative specifications under different assumptions regarding the

[6] The use of the symmetric steady state is standard in the IRBC literature. See Baxter and Crucini (1995) and Kollmann (1996).

degree of increasing returns-to-scale and driving source of economic fluctuations. The first model is a standard IRBC economy with complete markets, perfect competition, and a constant returns-to-scale technology, which corresponds to the special case with $\lambda = 1$ in the model we have described. We will refer to this configuration as the RBC benchmark. The assumption of complete markets allows for perfect risk pooling, and therefore implies in the symmetric case we are considering that

$$c_{1t} = c_{2t}, \text{ for all } t, \tag{21}$$

which once substituted into (14)–(17) provides the dynamic system for this specification. It follows that capital allocations across countries are given by:

$$K_{1t} = \frac{K_t}{1 + Z_{1t}^{p} Z_{2t}^{-p}}, \text{ and } K_{2t} = \frac{K_t}{1 + Z_{1t}^{-p} Z_{2t}^{p}}. \tag{22}$$

The propagation of international business cycles in this economy can be clearly described in terms of effects to the labor markets in the home and foreign economies. A positive domestic productivity shock shifts the labor demand curve outwards, increasing the real wage and employment. This, in turn, increases the marginal product of capital and therefore capital inflows that strengthen the employment and output effects. Abroad, the outflow of capital will reduce the demand for labor, lowering employment and thus output. Consequently, output levels are likely to be *negatively* correlated across countries, while consumptions remain *positively* correlated because agents have diversified away all country-specific risk. This negative transmission of business cycles is the mechanism through which the consumption-output anomaly arises in the RBC benchmark.

3.2. *The Incomplete Markets Version.* This specification maintains the assumptions of perfect competition and constant returns-to-scale, but rather than allowing for trade in a complete set of contingent claims, the agent only has access to a one-period discounted risk-free bond b_t. Thus, his (her) budget constraint in country i becomes:

$$c_{it} + K_{it+1} - (1-\delta)K_{it} + \frac{b_{it+1}}{1 + r_{bt}} = w_{it}L_{it} + r_t K_{it} + b_{it}, \tag{23}$$

where r_{bt} is the world rate of return on risk-free bonds at time t. We also have to impose the transversality condition:

$$\lim_{t \to \infty} \rho^t \frac{b_{it+1}}{(1 + r_{bt})^t} = 0. \tag{24}$$

The first-order conditions for this optimization problem are equations (11)–(12), together with

$$c_{it}^{-\theta} = \rho(1 + r_{bt}) E_t\left[c_{it+1}^{-\theta}\right]. \tag{25}$$

In addition, the equilibrium condition for the world bonds market is given by $b_{1t} + b_{2t} = 0$, which implies that the world goods market also clears because of Walras' Law. From equations (11)–(13), together with (23) and (25), it follows that the dynamic system is identical to that of equations (14)–(17), and international capital allocation is given by (18).

For the computation of the steady state, we follow Baxter and Crucini (1995) by setting $b^*/Y^* = 0$ for both countries. That is, the steady-state level of asset holdings are equal to zero so that per-capita wealth is equal across countries. This implies that $c_1^* = c_2^*$, which not surprisingly corresponds to the same steady state as in equation (20).

Since closing international assets markets forces agents to bear nation-specific risk, we expect that the cross-country consumption correlation in this specification to be less than one. However, our simulation results show that it is not much lower, mainly because productivity shocks are highly correlated across countries. In other words, restrictions on asset trade alone appear unable to resolve the consumption-output anomaly.

3.3. *The Baxter-King Version.* In this specification, we modify the RBC benchmark and the incomplete markets version by introducing increasing returns into the production technology, as in Baxter and King (1991). The degree of increasing returns postulated in this framework is small enough to preserve saddle path stability. Baxter and King (1991) use externalities to reconcile aggregate increasing returns with competitive factor markets; however, as explained in Section 2, we assume that output is produced from a continuum of intermediate inputs, each of which is produced by a monopolistic competitor. Another difference is that their economy is subject to preference shocks, whereas our model is driven by productivity disturbances.

This specification (either in its complete markets or incomplete markets version) is qualitatively similar to the CRS counterpart, except that labor demand elasticity is higher in this formulation. Thus, the effects of international business cycle transmission described for the RBC benchmark will be strengthened. This implies that the combination of incomplete markets and increasing returns cannot account for the consumption-output anomaly, as long as the steady state remains as a saddle point.

3.4. *Expectational Shocks.* Benhabib and Farmer (1994) have shown that if increasing returns are strong enough, the 'equilibrium labor demand schedule' in (11) may slope up as a function of real wage, and is steeper than the labor supply curve. In this case, the steady-state dynamics will be changed from a saddle to a sink.[7] It follows that in this economy, the consumption Euler equation for the

[7] Taking the logarithm on both sides of equation (11) indicates that the slope of the equilibrium labor demand schedule is given by $\beta - 1$, and that the slope of the labor supply equals $-\gamma$. Benhabib and Farmer (1994) have shown that in the continuous time framework, the necessary and sufficient conditions for indeterminacy are $\beta - 1 > 0$, and $\beta - 1 > -\gamma$. In the discrete time version of the model, these are necessary but not sufficient conditions.

representative household in country i is:

$$c_{it}^{-\theta} = \rho E_t \left\{ c_{it+1}^{-\theta} \left(1 - \delta + a \frac{Y_{it+1}}{K_{it+1}} \right) + V_{it+1} \right\},\ i = 1,2, \tag{26}$$

where we have shut down the productivity shocks completely. The term V_{t+1} represents any random variable with zero conditional mean at time t, which can be interpreted as the self-fulfilling beliefs of investors. In the previous configurations with saddle point dynamics, the error term to equation (26) is linked to the fundamentals of the economy by the cross-equation restrictions that place the economy on the stable branch of the saddle path. That is, sunspots do not matter in those economies. However, in the current setup with an indeterminate steady state, it is no longer possible to uniquely pin down agents' beliefs as a function of economic fundamentals. Thus, the disturbance term V can be an independent source of economic fluctuations. In this case, it is plausible for the economy to display belief-driven cycles in the absence of any underlying fundamental uncertainty.

While there are no fundamental disturbances, capital accumulation will react to the belief shocks through the consumer's budget constraint. Therefore, international financial markets still play an important role in insuring against capital movements, and agents will hold, as before, one half of the equities in each country. Given the assumption that there are no markets for claims contingent on sunspot realizations, shocks to the consumption Euler equations cannot be insured away, hence divergent consumption patterns may arise.

The dynamic system for this economy is derived from assuming away the productivity disturbances in (14)–(17), substituting the consumption Euler equations by (26), and replacing the process for the technology shocks with one describing the evolution of beliefs. Consequently, the dynamic system is given by:

$$\begin{aligned} K_{t+1} = {} & (1-\delta)K_t + AK_t^{\phi} c_{1t}^{d}(1 + c_{1t}^{s} c_{2t}^{-s})^{-\phi} \\ & + AK_t^{\phi} c_{2t}^{d}(1 + c_{1t}^{-s} c_{2t}^{s})^{-\phi} - c_{1t} - c_{2t}, \end{aligned} \tag{27}$$

$$c_{1t}^{-\theta} = E_t\left\{ BK_{t+1}^{\phi-1} c_{1t+1}^{d-\theta} (1 + c_{1t+1}^{s} c_{2t+1}^{-s})^{-\phi+1} + \tau c_{1t+1}^{-\theta} + V_{1t+1} \right\}, \tag{28}$$

$$c_{2t}^{-\theta} = E_t\left\{ BK_{t+1}^{\phi-1} c_{2t+1}^{d-\theta} (1 + c_{1t+1}^{-s} c_{2t+1}^{s})^{-\phi+1} + \tau c_{2t+1}^{-\theta} + V_{2t+1} \right\}, \tag{29}$$

where all the parameters are defined in the same way as before.

For the symmetric case that we are considering, it is straightforward to show that the steady state of the above dynamic system is unique, and has the same expression as in (20). In addition, the allocation of capital to each country is:

$$K_{1t} = \frac{K_t}{1 + c_{1t}^{s} c_{2t}^{-s}},\ \text{and}\ K_{2t} = \frac{K_t}{1 + c_{1t}^{-s} c_{2t}^{s}}, \tag{30}$$

and employment in country i at period t is decided according to (19) with $Z_{it} = 1$.

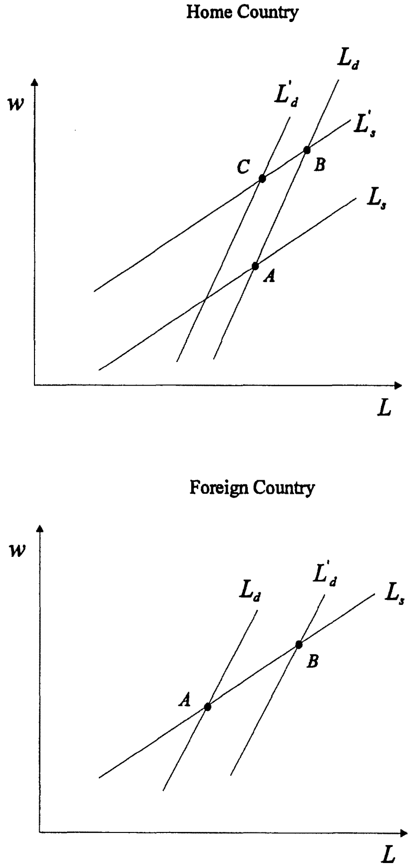


FIGURE 1

LABOR MARKETS OF HOME AND FOREIGN COUNTRIES IN THE EXPECTATIONAL SHOCKS MODEL

The intuition for how the expectational shocks model works is depicted in Figure 1, which shows the labor markets of the domestic and foreign economies. In this framework, in which the strength of increasing returns is sufficient to change the stability properties of the model, the equilibrium labor demand schedule slopes upward and is steeper than the labor supply curve. When the home economy is subject to a positive belief shock, the lower marginal utility of current consumption shifts the labor supply curve L_s upwards to L'_s. The resulting excess demand for labor moves the equilibrium from A to B, which increases employment and the wage rate. This leads to an increase in the marginal product of capital at home, inducing a capital inflow into the domestic economy. As a consequence, the domestic equilibrium labor demand schedule L_d shifts upwards to L'_d, generating an excess labor supply that tends to offset, to some extent, the initial effect on employment, and moves the equilibrium to point C. While theoretically we cannot determine whether labor hours increase or not in the home economy, they do increase in our simulations. Abroad, the capital outflow shifts the demand for labor downward from L_d to L'_d. This, in equilibrium, yields an increase in employment (from A to B) because the strong increasing returns exponentiate the productivity decline to the point that agents prefer to substitute leisure for labor. As a result, outputs are *positively* correlated across countries, whereas consumptions are likely to respond weakly to these transitory belief shocks.

4. MODEL CALIBRATION

4.1. *Benchmark Parameter Values.* To derive the linear approximations, solve for the equilibrium, and obtain model-generated time series, we follow the tradition of the real business cycle paradigm by assigning specific numerical values to the parameters of the model based on evidence from growth observations, panel studies of individual households, and empirical investigation in the industrial organization literature.

The labor supply elasticity of the representative household is set to be 4, that is, $\gamma = -0.25$, a number adopted by King et al. (1988). We also use a quarterly discount factor $\rho = 0.99$, and a quarterly depreciation rate $\delta = 0.025$, both of which are standard in the RBC literature. In addition, we follow Backus et al. (1992) by choosing the parameter governing the intertemporal elasticity of substitution, θ, to be 2.

The parameter of labor share in national income b is chosen to be 0.70, a figure within the range found by Christiano (1988). In the first two economies (RBC benchmark and incomplete markets version) with $\lambda = 1$, factor shares in total output, a and b, are equal to their respective elasticities in production, α and β. However, in the models with increasing returns, these parameters differ since $a = \lambda\alpha$ and $b = \lambda\beta$, where λ measures the monopoly power of intermediate input producers. To get a fix on the value of this parameter, we find that $1/\lambda$ is equal to the mark-up of price over marginal cost. In recent studies of U.S. manufacturing industries, Basu and Fernald (1994) present estimates of value-added mark-ups of at most 1.2, and Morrison (1990) shows that the same mark-up ranges from 1.2 to 1.4 for sixteen out of her eighteen industries, Hall's (1986) estimate lies above 1.4 for seventeen out of his twenty-eight industries; whereas Domowitz et al. (1988) estimate gross output mark-ups in the order of 1.4 to 1.7 for seventeen out of their nineteen industries. Drawing on these studies, we use a price-cost mark-up of 1.30 for the Baxter and King versions, and of 1.91 for the expectational shocks model. These values in turn imply that $\lambda = 0.769$ and $\lambda = 0.525$, respectively.

Another key parameter to be calibrated is the capital share of total output, a. In the CRS specifications with perfect competition, $a + b$ is equal to 1. In the IRS economies, we arbitrarily set the monopoly profits to 6% of national income, that is, $a + b = 0.94$.[8] It follows that a is equal to 0.24 in these models. The values of

[8] The simulation results in the following section are not sensitive to the choice of profit share in national income. We have explored several parameterizations of monopoly profits between 2 and 8%, and we obtain similar results in all cases.

TABLE 1
BENCHMARK PARAMETER VALUES

Model	λ	a	α	b	β	Returns to scale
RBC Benchmark	1.00	0.30	0.30	0.70	0.70	1.00
Incomplete Markets	1.00	0.30	0.30	0.70	0.70	1.00
Baxter-King with Complete Markets	0.77	0.24	0.31	0.70	0.91	1.22
Baxter-King with Incomplete Markets	0.77	0.24	0.31	0.70	0.91	1.22
Expectational Shocks	0.53	0.24	0.46	0.70	1.33	1.79

a, b, α, β and returns-to-scale of aggregate production function in each model implied by the above discussion are summarized in Table 1.[9]

Notice that the expectational shocks model differs from the other economies since the degree of increasing returns is large enough to cause the equilibrium labor demand schedule to become upward sloping, that is, the parameter β is greater than 1. This implies that to satisfy the condition for indeterminacy in the current one-commodity world economy, the required value of mark-up or degree of externalities must be at the high end of the empirically reasonable range. However, this is certainly not a necessary feature of the multiple equilibria literature. Recently, Benhabib and Farmer (1996) and Perli (1997) have shown that a two-sector closed economy version of the RBC model with a conventional, downward sloping labor demand curve may display indeterminacies for much smaller magnitudes of increasing returns than those suggested by previous studies.

4.2. *Driving Processes.* Our first four specifications are driven by productivity shocks. We calibrate the bivariate technology shock process (17) using estimates of Solow residuals constructed by:[10]

$$Z_{it} = \frac{Y_{it}}{L_{it}^{\beta}},\ i = 1, 2, \tag{31}$$

where Y is output, and L is employment for U.S. and an European aggregate, 1970:1–1990:3.[11] Before estimating equation (17), we normalize each estimate of Z to have a sample mean of one. To summarize the quantitative properties of the models by reporting statistics for a single country, we follow Backus et al. (1992) by using a symmetric version of estimates from the U.S.-Europe system, which we present in Table 2.

In the expectational shocks economy, animal spirits substitute for the technology shock as the only driving force behind international business cycles. We thus set $Z_{1t} = Z_{2t} = 1$ for all t. We calibrate the belief shocks in the consumption Euler

[9] In general, the degree of returns-to-scale in production is $(a+b)/\lambda$, which is equal to $\alpha + \beta$.

[10] As pointed out by Backus et al. (1992), variations in the capital input are ignored since comparable capital stock data are not available on a quarterly basis. This is probably not a serious problem because capital stock contributes very little to the output fluctuations. See Kydland and Prescott (1982).

[11] We thank David Backus for providing us the data set used in Backus et al. (1995).

TABLE 2
TECHNOLOGY SHOCK PROCESS

CRS specifications	Baxter-King specifications
$\Omega = \begin{bmatrix} 0.940 & 0.058 \\ 0.058 & 0.940 \end{bmatrix}$	$\Omega = \begin{bmatrix} 0.947 & 0.047 \\ 0.047 & 0.947 \end{bmatrix}$
$\text{var}(\eta_1) = \text{var}(\eta_2) = 0.00862^2$	$\text{var}(\eta_1) = \text{var}(\eta_2) = 0.00923^2$
$\text{corr}(\eta_1, \eta_2) = 0.306$	$\text{corr}(\eta_1, \eta_2) = 0.289$

equations (28) and (29) as *innovations* to the Index of Consumer Sentiment for the U.S., and to the Harmonized Consumer Survey for Europe. These disturbances represent shocks to 'consumer confidence' (CC) as they measure expectations about the future evolution of the economy.[12] A higher value of CC indicates that people are more optimistic about the future, and therefore they will reduce the value assigned to consumption next period. This, in turn, affects the consumption decisions today in the same way as a preference shock that reduces future utility. To obtain time series for these innovations, we estimate a bivariate autoregressive process using the CC data for the U.S. and an European aggregate, denoted as CC_{US} and CC_{Eu}, respectively:

$$(32) \qquad \begin{bmatrix} CC_{USt} \\ CC_{Eut} \end{bmatrix} = \begin{bmatrix} 0.703(.20) & -0.103(.67) \\ 0.089(.04) & 0.612(.14) \end{bmatrix} \begin{bmatrix} CC_{USt-1} \\ CC_{Eut-1} \end{bmatrix} + \begin{bmatrix} V_{1t} \\ V_{2t} \end{bmatrix},$$

where the numbers in parentheses are standard errors, and V_1 and V_2 are the sunspot shocks used in subsequent simulations.[13] Appendix Section A.1 describes how these CC indices are constructed from the original survey data. In particular, the data used in this paper are a recomputed series that homogenizes the U.S. and European indices. While we have chosen to work with a proxy for the expectational shocks that is based on real world data, any random variable with a zero conditional mean would serve the purpose. In fact, following Farmer and Guo (1994), we have rescaled Σ, the variance-covariance matrix of V_1 and V_2 in (32), so that the model-generated output variabilities that match those of the U.S. economy.[14] This results in a rescaled matrix:

$$(33) \qquad \Sigma^* = k\Sigma = k\begin{pmatrix} 0.00242 & 0.00109 \\ 0.00109 & 0.00242 \end{pmatrix},$$

[12] For evidence that output fluctuations can be caused by expectational shocks, see Matsusaka and Sbordone (1995). They report that consumer confidence accounts for between 13 and 26 per cent of the innovation variance of U.S. GNP.

[13] This estimated matrix has a pair of complex eigenvalues, $0.607 \pm 0.084i$. Notice that the CC indices are less persistent than the technology shocks obtained from Solow residuals.

[14] In other words, we have calibrated a belief process that leads to an equilibrium in which the standard deviation of the artificial output series is equal to that of actual data. See Farmer (1993) on the related issue of equilibrium selection in models with multiple equilibria.

where $k = 1/6000$ is the rescaling parameter. Notice that the correlation between V_1 and V_2 is equal to 0.45, which is stronger than that of the innovations to technology shocks shown in Table 2.

5. SIMULATION RESULTS

Table 3 summarizes the business cycle properties of the above five model economies, together with those of the international data taken from Backus et al. (1992). To facilitate comparison with earlier work, we also present the findings of the Backus et al. (1992) One-Quarter Time to Build specification. See Appendix Section A.2 for details on the solution procedure for each configuration.

Among the models that are driven by productivity shocks, the RBC benchmark is most comparable to the Backus et al. (1992) One-Quarter Time to Build specification. However, there are differences in preferences and production (they have an inventory component) that require checking whether we have provided a framework that is comparable to traditional IRBC models with a nonseparable utility function. A comparison of the One-Quarter Time to Build model and our RBC benchmark indicates that both versions exhibit very similar *qualitative* properties. They are both subject to the well-known flaws found in the IRBC literature: a procyclical and volatile current account, high investment fluctuations, low investment and savings correlation, and the consumption-output anomaly. Procyclical international capital

TABLE 3
SIMULATION RESULTS OF ALTERNATIVE ECONOMIES

					Baxter-King		
	Data	BKK	RBC Benchmark	Incomplete Markets	Complete markets	Incomplete markets	Expectational Shocks
Standard deviations							
σ_y	1.71	2.24	2.03	1.96	5.42	6.20	1.72*
$\sigma_{NX/y}$	0.45	8.78	10.15	0.15	33.01	0.40	0.90
Standard deviations relative to output							
σ_c/σ_y	0.49	0.30	0.12	0.13	0.07	0.04	0.05
σ_I/σ_y	3.15	31.47	30.12	33.06	31.97	33.79	6.46
σ_L/σ_y	0.86	—	0.79	0.79	0.80	0.85	0.77
Contemporaneous correlations with output							
$\rho(c, y)$	0.76	0.76	0.17	0.31	0.05	−0.07	0.43
$\rho(I, y)$	0.90	−0.01	−0.37	−0.38	−0.38	−0.38	0.86
$\rho(L, y)$	0.86	—	0.97	0.97	0.99	0.99	0.99
$\rho(NX/y, y)$	−0.28	0.11	0.49	0.48	0.52	0.51	−0.009
International contemporaneous cross correlations							
$\rho(y, y^*)$	*0.70*	−0.58	−0.94	−0.92	−0.99	−0.99	*0.98*
$\rho(c, c^*)$	*0.46*	0.69	1.00	0.97	1.00	0.96	*0.44*
$\rho(I, S)$	0.69	−0.01	−0.39	−0.38	−0.38	−0.38	0.87

The first column is taken from Backus et al., (1992). BKK refers to their One-Quarter Time to Build model. (*) indicates that this number is not estimated.

movements responding to productivity shocks generate negative cross-country output correlation, and a low domestic investment-savings correlation.

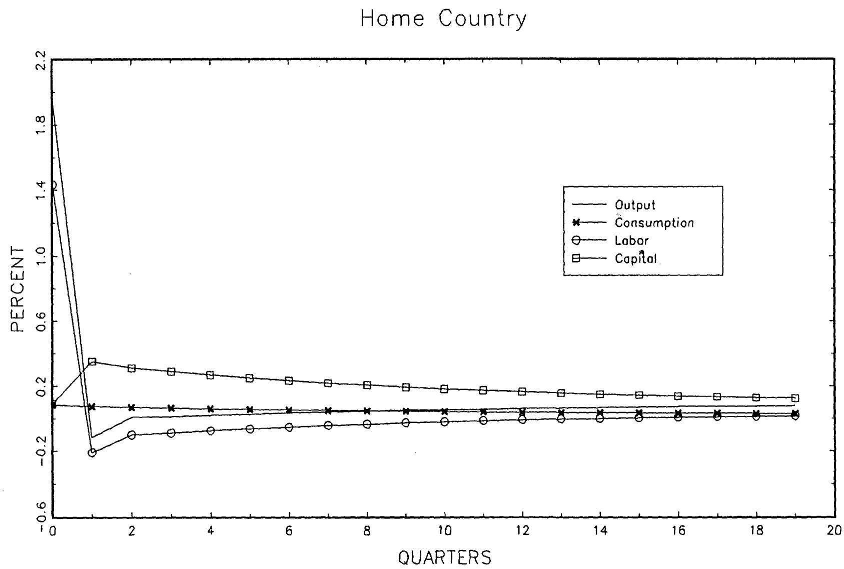
In the Baxter and King version with complete markets, the output variance increases, and becomes several times larger than that in the data. Furthermore, this economy seems to worsen the problems of the RBC benchmark because the higher elasticity of labor demand magnifies the effects of capital movement, increasing the variability of investment and the current account. Since the international output correlation turns out to be even smaller than that in the RBC benchmark, and consumption correlation across countries still equals one, this model cannot resolve the consumption-output anomaly.

The same drawbacks characterize the two specifications with incomplete markets. In particular, the consumption-output anomaly and the low savings-investment correlation still persist. Due to the spillover effects of technology shocks across countries (see Table 2), all fluctuations in productivity shocks are almost common to each country. This implies that there is little room to share risk in the first place. As a result, international consumption correlation remains high, and in general, incomplete asset markets have negligible effects on the international business cycle properties.[15]

However, the expectational shocks model appears to do a better job in accounting for the consumption-output anomaly: output levels are strongly correlated while consumption levels are only weakly so. Figure 2 presents the dynamic responses of output, consumption, labor and capital to a one-standard-deviation belief shock in the home economy. Outputs move strongly together because labor supply responds in the same direction in both countries: at home due to the positive belief shock, and abroad because capital outflows induce more labor hours. The impulse response functions in this figure illustrate that the convergence of various macroeconomic variables to the stationary state is slow with the domestic capital stock staying above its steady-state value throughout the transition period. Notice, however, that employment in the home economy falls below the steady state in the second period and only gradually returns to it. This is exactly what we should expect from our discussion of Figure 1, when labor supply moves back to its original position, but in an economy which is now endowed with a larger capital stock (that is, where the domestic labor demand curve is at L'_d). An opposite adjustment process takes place abroad. The response of consumption is small because shocks are transitory, and permanent income remains virtually unchanged. The belief shock generates a small consumption response at home, and the foreign consumption barely moves, leading to a relatively weak cross-country consumption correlation.

To check the robustness of these results, Table 4 presents simulation results for alternative configurations of the expectational shocks model. Notice that international output and consumption correlations are not affected by changes in the

[15] Baxter and Crucini (1995) present an alternative specification with random walk technology shocks, which generates strong wealth effects to any change in productivity such that the consumption-output anomaly is resolved. However, when they use the more standard trend stationary technological innovation considered in this paper, their result exhibits the same flaws as in our CRS specification, either with complete or incomplete markets.



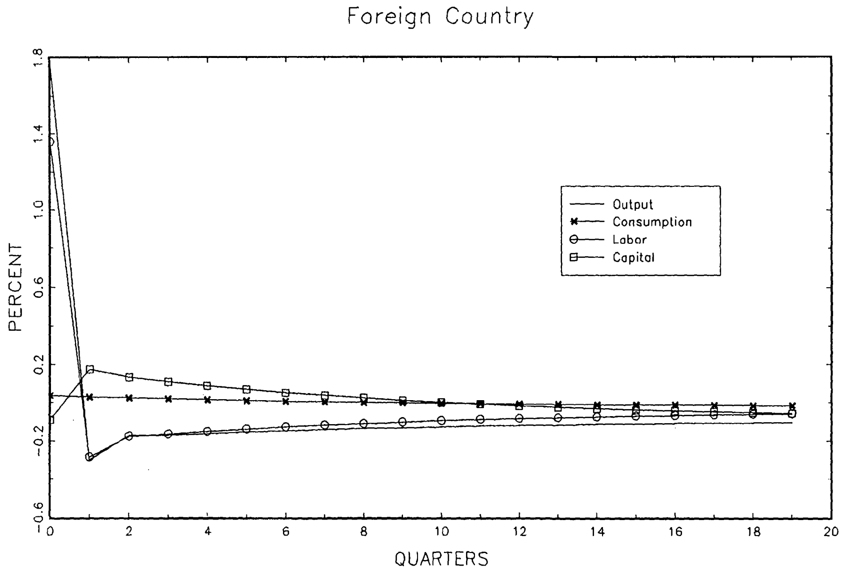


FIGURE 2

IMPULSE RESPONSE FUNCTIONS IN THE EXPECTATIONAL SHOCKS MODEL

TABLE 4

ROBUSTNESS OF THE EXPECTATIONAL SHOCKS MODEL

Statistics	Data	$\gamma = -0.5$	$\gamma = 0$	$\theta = 1$	$\theta = 3$
$\rho(y, y^*)$	0.70	0.98	0.91	0.98	0.98
$\rho(c, c^*)$	0.46	0.48	0.68	0.45	0.44
$\rho(I, S)$	0.69	0.85	0.55	0.86	0.87

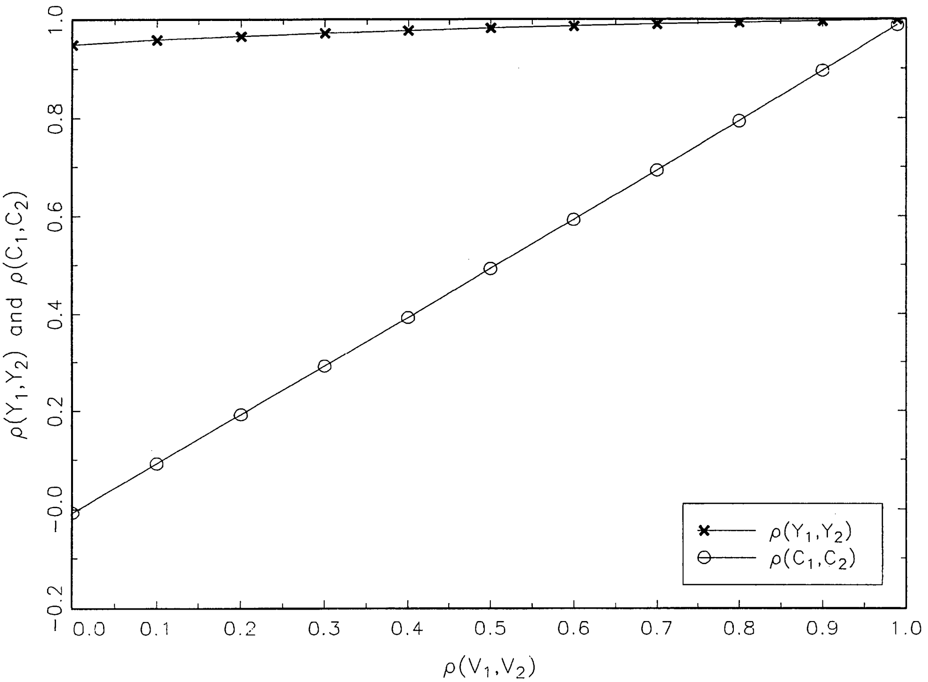


FIGURE 3

INTERNATIONAL OUTPUT AND CONSUMPTION CORRELATIONS IN THE EXPECTATIONAL SHOCKS MODEL

coefficient of relative risk aversion, θ, and variations of the labor supply elasticity parameter, γ. The values of $\theta = 1$ correspond to the setup with a logarithmic utility function in consumption, and $\theta = 3$ lies within the range of $[0.5, 3]$ estimated by Eichenbaum et al. (1988). The value $\gamma = -0.50$ corresponds to that used in Backus et al. (1992), and the specification of *indivisible labor* with $\gamma = 0$ draws on the studies by Hansen (1985) and Rogerson (1988), which have been shown to provide a better fit of labor market business cycle statistics for the closed economy.[16]

The cross-country consumption correlation, however, is quite sensitive to the correlation coefficient between V_1 and V_2, which is denoted by ρ_v. Figure 3 plots how output and consumption correlations between countries relate to this cross correlation of shocks. The figure shows clearly that these belief shocks do not have to be excessively correlated across countries to account for the consumption-output anomaly. Our results thus illustrate that not only are incomplete markets required to explain the consumption-output anomaly, but that the driving process of the economy cannot have very high correlation across countries if it is to be consistent with the data.

[16] To continue satisfying the conditions for indeterminacy $\beta > 1$, and $\beta - 1 > -\gamma$ in the case when the labor supply elasticity is reduced from 4 ($\gamma = -0.25$) to 2 ($\gamma = -0.5$), we lower the value of λ to be 0.43 in the simulations.

6. CONCLUSION

Previous literature indicates that it is difficult to explain most of the relevant characteristics of international business cycles in a model driven by technology shocks. In an economy with a separable utility function, complete contingent claims markets lead to a perfect pooling of risks, and therefore perfect cross-country consumption correlation. In addition, investment is highly variable because capital moves to the country with positive productivity disturbances. This, in turn, induces a low international output correlation. With incomplete markets, the same problem persists because productivity shocks tend to be strongly correlated across countries. This paper shows that when increasing returns are strong enough to make the equilibrium indeterminate, a belief-driven model can provide a good fit of international business cycle statistics, and is able to account for the consumption-output anomaly under the assumption that there are no contingent claim markets on the realizations of sunspot shocks.

Since our model relies on self-fulfilling belief shocks that influence agents' desire to consume and elicit strong labor supply response, we interpret our result as confirming the suggestion proposed by Stockman and Tesar (1995): demand shocks are important in understanding the properties of international business cycles. In our specification, however, rather than incorporating demand effects from assuming an ad-hoc labor market or from exogenous preference shocks, we obtain the results from a well-specified dynamic general equilibrium model with calibrated disturbances, flexible prices and clearing markets.

We started this project by trying to assess the implications of introducing increasing returns into a standard IRBC economy. Since internal increasing returns-to-scale to firms are solidly ingrained in international trade theory, we believe that this element should be taken into account in IRBC studies as well. What we have concluded from our exercise is that as long as increasing returns do not change the *dynamics* of the model, they tend to aggravate the problems inherent in the traditional CRS specification. Nevertheless, a model with an indeterminate steady state, subject to belief shocks, seems to have a better chance at explaining the international data than earlier studies driven by productivity disturbances. We read this finding not so much as a defense of 'crazy explanations' of international business cycles, but as very suggestive of the improvement in fit obtained by using disturbances that affect the demand side of the economy. Since these demand shocks might be related to fiscal and monetary policies and their transmission mechanism, further research in this direction should be the next step.

7. APPENDIX

A.1. *Consumer Confidence Data*. The U.S. consumer confidence data is taken from the Index of Consumer Sentiment collected by the Survey Research Center at University of Michigan. The monthly survey is based on 500 telephone interviews, and utilizes a rotating panel sample design. Five questions listed in Table 5 are asked during the interview. Quarterly data are available from the first quarter of 1960.

TABLE 5
CONSUMER CONFIDENCE SURVEYS

United States questions:

(1) Would you say that you are better off or worse off financially than you were a year ago?
(2) Looking ahead, do you think that a year from now, you will be better off financially, or worse off, or just about the same as now?
(3) Do you think that during the next twelve months we'll have good times financially, or bad times, or what?
(4) Looking ahead, which would you say is more likely, that in the country as a whole we'll have continuous good times during the next five years or so, or that we will have periods of widespread unemployment or depression, or what? [dropped]
(5) Generally speaking, do you think now is a good time or bad time for people to buy major household items?

European questions:

(1) How does the financial situation of your household now compare with what it was 12 months ago?
(2) How do you think the financial position of your household will change over the next 12 months?
(3) How do you think the general economic situation in this country has changed over the last 12 months? [dropped]
(4) How do you think the general economic situation in this country will develop over the next 12 months? [matched with U.S. question (3)]
(5) Do you think there is an advantage for people to make major purchases at the present time?

The European data is taken from the Harmonized Consumer Survey, published by the commission of the European Community (EC). The monthly survey is based on 2000 interviews in each country (1250 in Ireland), some by phone and some in person. Respondents are asked five questions, which are also listed in Table 5. Results are added across member countries of the EC. Monthly data are available since the third quarter of 1986, and is transformed into quarterly averages. Three issues of data compatibility arise:

(1) Only four of the questions are consistent between the U.S. and European samples. Therefore, one question is dropped from each data set and the indexes are recalculated.
(2) The U.S. figures are reported in index form, while the European figures are the raw scores. Thus, we transform the European numbers to be indexed as is the U.S. data.
(3) For each of the European questions, five possible answers are available, ranging from 'a lot worse' to 'a lot better;' whereas each of the U.S. questions has only three possible responses, ranging from 'worse' to 'better.' (The one exception is the question regarding purchasing major household items, for which both surveys have only three possible responses.) This has not been corrected.

A.2. *Solution Procedures*. The dynamics of models with incomplete markets are described by the nonlinear functional equations (14)–(17). For the specifications with complete markets, $c_{1t} = c_{2t}$ is incorporated into the dynamic system. Moreover, the expectational shocks model is represented by equations (27)–(29). The state vector is $\{K_t, c_{1t}, c_{2t}, Z_{1t}, Z_{2t}\}$ for the incomplete markets setup, $\{K_t, c_{1t}, Z_{1t}, Z_{2t}\}$

for the complete markets formulation, and $\{K_t, c_{1t}, c_{2t}\}$ for the expectational shocks model, respectively. To analyze the dynamic properties of alternative economies, first the deterministic steady state in each version of the model is derived using equation (20). We then adopt the approach of King et al. (1988) by taking log-linear approximations to the dynamic systems in the neighborhood of their respective steady states. For the incomplete markets cases, a linear approximation to equations (14)–(17) can be written as:

$$(34)\quad \begin{bmatrix} \hat{K}_t \\ \hat{c}_{1t} \\ \hat{c}_{2t} \\ \hat{Z}_{1t} \\ \hat{Z}_{2t} \end{bmatrix} = J_1 \begin{bmatrix} \hat{K}_{t+1} \\ \hat{c}_{1t+1} \\ \hat{c}_{2t+1} \\ \hat{Z}_{1t+1} \\ \hat{Z}_{2t+1} \end{bmatrix} + R_1 \begin{bmatrix} E_t[\hat{K}_{t+1}] - \hat{K}_{t+1} \\ E_t[\hat{c}_{1t+1}] - \hat{c}_{1t+1} \\ E_t[\hat{c}_{2t+1}] - \hat{c}_{2t+1} \\ \hat{\eta}_{1t+1} \\ \hat{\eta}_{2t+1} \end{bmatrix}, \hat{K}_0, \hat{Z}_{10}, \hat{Z}_{20} \text{ are given.}$$

In the complete markets specifications with $c_{1t} = c_{2t}$, the linearized dynamic system is given by:

$$(35)\quad \begin{bmatrix} \hat{K}_t \\ \hat{c}_{1t} \\ \hat{Z}_{1t} \\ \hat{Z}_{2t} \end{bmatrix} = J_2 \begin{bmatrix} \hat{K}_{t+1} \\ \hat{c}_{1t+1} \\ \hat{Z}_{1t+1} \\ \hat{Z}_{2t+1} \end{bmatrix} + R_2 \begin{bmatrix} E_t[\hat{K}_{t+1}] - \hat{K}_{t+1} \\ E_t[\hat{c}_{1t+1}] - \hat{c}_{1t+1} \\ \hat{\eta}_{1t+1} \\ \hat{\eta}_{2t+1} \end{bmatrix}, \hat{K}_0, \hat{Z}_{10}, \hat{Z}_{20} \text{ are given.}$$

Finally, for the expectational shocks model, equations (27)–(29) are approximated by:

$$(36)\quad \begin{bmatrix} \hat{K}_t \\ \hat{c}_{1t} \\ \hat{c}_{2t} \end{bmatrix} = J_3 \begin{bmatrix} \hat{K}_{t+1} \\ \hat{c}_{1t+1} \\ \hat{c}_{2t+1} \end{bmatrix} + R_3 \begin{bmatrix} E_t[\hat{K}_{t+1}] - \hat{K}_{t+1} \\ E_t[\hat{c}_{1t+1}] - \hat{c}_{1t+1} \\ E_t[\hat{c}_{2t+1}] - \hat{c}_{2t+1} \end{bmatrix}, \hat{K}_0 \text{ is given.}$$

In (34)–(36), 'hat' variables denote per cent deviations from their steady-state values, J_1, J_2, and J_3 are the Jacobian matrices of partial derivatives of the transformed dynamic systems, and R_1, R_2, and R_3 are conformable matrices of coefficients.

The stability of the stationary equilibrium in (34)–(36) is determined by comparing the number of eigenvalues of J_1, J_2, and J_3 located outside the unit circle with the number of initial conditions in each linearized dynamic system. For models characterized by saddle-point dynamics, as in (34) and (35), J_1 and J_2 both exhibit three roots outside the unit cycle. To find the unique rational expectations solution to (34), we iterate the two 'stable' roots (inside the unit cycle) of J_1 forward to obtain the stable branch of the saddle path, which expresses $\hat{c}_{1t}$ and $\hat{c}_{2t}$ as linear functions of $\hat{K}_t$, $\hat{Z}_{1t}$, and $\hat{Z}_{2t}$. Similarly, iterating the only stable root of J_2 forward yields the unique stationary equilibrium trajectory in (35).

In the expectational shocks model with strong increasing returns, the eigenvalues of J_3 may all be outside the unit cycle. This implies that the steady state is indeterminate and thus a sink. To simulate this model economy, we take the inverse of J_3, and solve the dynamic systems backwards. As a result, equation (36) has the following representation:

$$\hat{K}_t = a_{11}\hat{K}_{t-1} + a_{12}\hat{c}_{1t-1} + a_{13}\hat{c}_{2t-1}, \tag{37}$$

$$\hat{c}_{1t} = a_{21}\hat{K}_{t-1} + a_{22}\hat{c}_{1t-1} + a_{23}\hat{c}_{2t-1} + b_1\hat{V}_{1t},$$

$$\hat{c}_{2t} = a_{31}\hat{K}_{t-1} + a_{32}\hat{c}_{1t-1} + a_{33}\hat{c}_{2t-1} + b_2\hat{V}_{2t},$$

where V_{1t} and V_{2t} are sunspot shocks that exhibit the covariance properties described in (33).

In each model, additional linear equations relating output, investment, labor hour, savings and net exports to the current state vector can be easily obtained. The statistics reported in Tables 3 and 4 are sample means computed for 100 simulations, each of which consists of 50 periods. All entries have been passed through the Hodrick-Prescott filter.

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